

STOCHASTIC VOLATILITY MODELS FOR FINANCIAL TIME SERIES ANALYSIS

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Abstract: *This article highlights a comprehensive and approachable perspective to stochastic volatility models for financial time series analysis. Financial time series represent a distinctive category in the economic field, with highly dynamic characteristics, especially in times of financial crisis. Beyond its highly empirical behavior, modeling volatility of financial asset returns aims to improve forecast accuracy. The stochastic volatility models analyzed in this article include the autoregressive conditional heteroscedastic model (ARCH), the generalized autoregressive conditional heteroscedastic (GARCH) model and the exponential generalized autoregressive conditional heteroscedastic (EGARCH) model.*

Key words: *stochastic volatility class of models, high-frequency data, stationary time series, autoregressive conditional heteroscedastic model, financial asset returns*

JEL classification: C22, G11, G14

1. Introduction

Modeling and forecasting the volatility of time series represents a stringent priority in terms of financial market analysis. Beyond the highly empirical behavior of financial time series, volatility is often perceived as a measure of risk. However, volatility is difficult to identify accurately and it is therefore to be estimated based on an econometric model.

Human understanding of financial markets is limited, because our capability in analyzing the time series is incomplete and empirical economic methods are not perfectly. The obvious complexity of the financial markets has been investigated by various researchers and a large amount of research papers has been published in recent past. Resolving such an applied complexity has been for many long years just an theoretical utopia (Birau, 2011b).

Financial time series are extremely unpredictable and non-stationary, generally characterized by periods in which registered local and non-systematic trends. As a consequence, the premise of a stochastic arbitrage probability is in sharp contradiction with the efficient market hypothesis (Franke, 2004).

Financial time series, such as daily financial asset returns are characterized by high-frequency and excessive volatility features known in the literature as “volatility clustering”. As a feature, financial asset returns exhibit an obvious extremely weak autocorrelation in the case of emerging capital markets, while they are completely uncorrelated in the case of developed capita markets. On the other hand, squared financial asset returns are strongly positively correlated.

The continuously-compounded daily returns were calculated using the log-difference, as follows:

$$r_t = \ln\left(\frac{p_t}{p_{t-1}}\right) = \ln(p_t) - \ln(p_{t-1})$$

where p was the daily closing price.

2. Stochastic volatility models

The univariate stochastic volatility models analyzed in this article include the autoregressive conditional heteroscedastic model (ARCH), the generalized autoregressive conditional heteroscedastic (GARCH) model and the exponential generalized autoregressive conditional heteroscedastic (EGARCH) model. Besides these models, volatility has many other applications which are based in part on mathematical finance and financial econometrics.

Financial time series exhibit periods of high volatility which alternate with periods of low volatility, as can be seen in the figure below that plots daily returns of BET index from January 2003 up to August 2012:

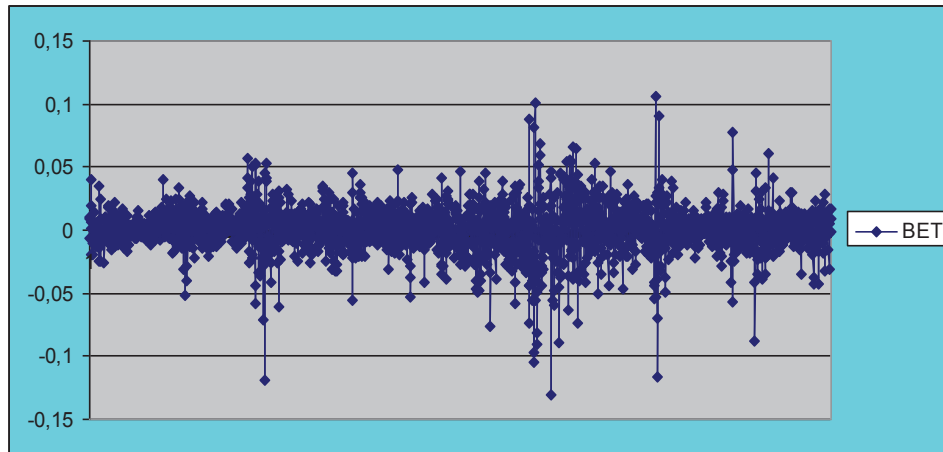


Fig.1 Plot of log returns of BET index

An important issue regarding the volatility process is that it focuses on the evolution of conditional variance of the financial asset return in time periods. Another key feature aimed the fact that volatility is a continuous process that does not involve major disparities. Moreover, volatility does not converge to infinity, but varies within certain fixed limits.

The Autoregressive Conditional Heteroskedasticity (ARCH) model was developed by Engle in 1982. The methodological innovation which sets it apart from the previous time series econometric models suggests that the variance of the error terms at the time moment t depends on the squared error terms from the past periods of time.

The essence of the model was that it is much more efficient to be used simultaneously the mean and variance of a financial time series in the case that conditional variance is not constant. The ARCH model highlights that the financial asset returns (r_t) depend on past information.

Setting F_t the informational set at time t which includes x_t and all other past realization of the process x_t , the financial asset returns is calculated as:

$$r_t = \mu_t + \varepsilon_t, \text{ where } E[\varepsilon_t / F_{t-1}] = 0$$

An ARCH (1) model is defined as the process ε_t , $t \in \mathbb{Z}$, if $E[\varepsilon_t / F_{t-1}] = 0$, then:

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2$$

In general, ARCH (q) model is defined as follows:

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \dots + \alpha_m \varepsilon_{t-m}^2$$

The generalized autoregressive conditional heteroscedastic (GARCH) model was developed by Bollerslev in 1986. The particular feature of this model was to

introduce and use the lagged conditional variance terms as autoregressive terms. Actually, GARCH (1,1) model highlights a parsimonious alternative option to a more complicated ARCH (q) model.

An GARCH (p,q) model is defined as the process ε_t , $t \in \mathbb{Z}$, if $E[\varepsilon_t / F_{t-1}] = 0$.

The variance equation is the following:

$$\sigma^2 = \alpha_0 + \sum_{i=1}^q \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^p \beta_j \sigma_{t-j}^2$$

The exponential generalized autoregressive conditional heteroscedastic (EGARCH) model was developed by Nelson in 1991.

The variance equation has the following expression:

$$\log \sigma_t^2 = \gamma_t + \sum_{k=1}^{\infty} \beta_k g(Z_t - k)$$

where γ_t and β_t are deterministic coefficients and simultaneously $E[g(Z_t)] = 0$.

The analysis highlights an important aspect that differentiates in a certain manner the last two stochastic volatility models, namely GARCH and EGARCH. Volatility in the case of GARCH model is an additive function of the lagged error terms ε_t , but on the other hand, volatility (conditional variance σ_t^2) is a multiplicative function of the lagged error terms ε_t .

3. Conclusions

Stochastic volatility models for financial time series analysis marks an area that has grown significantly in the recent past. Reliable modeling and forecasting of volatility is an extremely important area of concern in terms of financial markets. Continuous time stochastic volatility models analyzed in this article are characterized by high accuracy estimation. The use of ARCH, GARCH and EGARCH models in applied financial econometrics provides an improved perspective on risk management, portfolio optimization and financial decisions process.

BIBLIOGRAPHY

1. Birău, R. Efficient capital market, The Young Economists Journal, no.17, pp.15-19, 2011
2. Birău, F.R. International Scientific Conference "Advancements in the theory of economic decisions under risk and uncertainty conditions. Fuzzy systems in economics", 26th edition, Iași, 16.09.2011, Box-Jenkins Methodology vs. Fuzzy Logic in stock market modeling, publicat în volumul XIII al conferinței, Editura Tehnopress, Iasi, ISBN 978-973-702-863-1, pag. 124-133
3. Birău, F.R. An analysis of weak-form efficiency on the Bucharest Stock Exchange, Annals of University of Craiova - Economic Sciences Series, 2011, vol. 3, issue 39, ISSN 1223-365X pp. 194-205
4. Birău, F.R. Normal distribution assumption in terms of emerging capital markets. The case of Bucharest Stock Exchange, International Conference "Economic Knowledge And Performance", Timisoara, 2012

5. Bollerslev, T Common persistence in conditional variances, and Engle, Econometrica, Vol. 61, No. 1, 1993, 167-186 R.F.
6. Dickey, D.A. Stationarity Issues in Time Series Models, Statistics and Data Analysis, Philadelphia, 2005, paper 192-30.
7. Dwyer, G.P. Nonlinear Time Series and Financial Applications, Federal Reserve Bank of Atlanta, January 2003
8. Franke, J. Introduction to Statistics of Financial Markets, Springer Hardle, W. Publisher, 2004
Hafner, C.
9. E. Ghysels, A. Stochastic Volatility, CIRANO, Scientific Series, ISSN Harvey and E. 1198-8177
Renault
10. A. C. Harvey Estimation of an Asymmetric Stochastic Volatility Model N. Shephard for Asset Returns, Journal of Business & Economic Statistics, vol. 14, no. 4, 1996, pp. 429-434
11. R. Mahieu and An Empirical Application of Stochastic Volatility Models, P. C. Schotman Journal of Applied Econometrics, no13, 1998, pp.333-360
12. T. C. Mills and The Econometric Modelling of Financial Time Series, 3rd R.N. Markellos ed., Cambridge University Press, New York, 2008, ISBN-13 978-0-511-38103-4
13. J.F. Muzy, J. Modelling fluctuations of nancial time series: from cascade Delour, and E. process to stochastic volatility model, The European Bacry Physical Journal B, 2000, pp. 537-548
14. J. Nakajima Stochastic volatility model with leverage and asymmetrically heavy-tailed error using GH skew Student's t-distribution, Computational Statistics & Data Analysis Journal Vol. 56, Issue 11, 2012, pp. 3690-3704
15. S. Pellegrini, Financial Econometrics and SV models, 2007
A. Rodriguez
16. Rodriguez, J A powerful test for conditional heteroscedasticity for and Ruiz, E. financial time series with highly persistent volatilities, Statistica Sinica 15, 2005, pp.505-525
17. M. Sewell Characterization of Financial Time Series, UCL Department of Computer Science, 2011
18. Strozzi, F. Application of non-linear time series analysis techniques to high frequency currency exchange data, Liuc Papers n. 99, Serie Metodi quantitativi 14, Suppl. a gennaio 2002
19. R. S. Tsay Analysis of Financial Time Series, 2nd ed., John Wiley & Sons, Inc., Publication, 2005, ISBN-13 978-0-471-69074-0